Math 1050–4 Midterm Exam #3

April 17, 2015

- No notes, books, etc. are allowed.
- Answer the questions in the spaces provided.
- You have 50 minutes to complete this exam.
- **Good luck!**

Name: Answer Key uNID: _____

For #1–7, are the statements true or false?

1. (1 point) $\log_a(zw) = \log_a(z) + \log_a(w)$ — 1. True
2. (1 point) $a^x a^y = a^{xy}$ — $a^x a^y = a^{x+y}$ — 2. False
3. (1 point) $\log_a(z^w) = \log_a(z)^w$ — $\log_a(z^w) = w\log_a(z)$ — 3. False
4. (1 point) $(a^x)^y = a^{x+y}$ — $(a^x)^y = a^{xy}$ — 4. False
5. (1 point) $\log_a\left(\frac{z}{w}\right) = \frac{\log_a(z)}{\log_a(w)}$ — $\log_a\left(\frac{z}{w}\right) = \log_a(z) - \log_a(w)$ — 5. False
6. (1 point) $\frac{a^x}{a^y} = a^{x-y}$ — 6. True
7. (1 point) A linear equation in two variables ($ax + by = r$) has two solutions. — 7. False

   Three possibilities: zero, one, or infinitely many solutions

For #8–10, write the expression as a rational number in standard form.

8. (1 point) $3^{-500}3^{520}3^{-23}$

   $3^{-500+520-23} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

   8. $1/27$

9. (1 point) $\left(5^{\frac{7}{3}}\right)^{\frac{9}{7}}$

   $5^{\frac{7}{3}\left(\frac{9}{7}\right)} = 5^3 = 125$

   9. 125

10. (1 point) $100,000^{\frac{3}{5}}$

    $(10^5)^{3/5} = 10^3 = 1,000$

    10. 1,000

Points earned: _____
out of a possible 10 points

For #11–12, write the expression as a rational number in standard form.

11. (1 point) $\log_{10}\left(\frac{1}{1,000}\right)$

$= \log_{10}(10^{-3}) = -3$

11. -3

12. (1 point) $\log_4\left(\frac{1}{\sqrt[4]{64}}\right)$

$\log_4\left(\frac{1}{\sqrt[4]{4^3}}\right)$

$= \log_4\left(\frac{1}{4^{3/4}}\right)$

$= \log_4(4^{-3/4})$

$= -3/4$

12. $-3/4$

13. (1 point) Solve for x if $\log_8(x) = -2$. (Write your answer as a rational number in standard form.)

$x = 8^{-2}$

$x = \frac{1}{8^2}$

$x = \frac{1}{64}$

13. $1/64$

Points earned: _____
out of a possible 3 points

14. (1 point) Solve for x if $2e^x - 5 = 3$.

$$2e^x = 3 + 5$$
$$2e^x = 8$$
$$e^x = \frac{8}{2}$$
$$e^x = 4$$
$$x = \log_e(4)$$

14. $x = \log_e(4)$

15. (1 point) Solve for x if $e^{11-2x} = \frac{3}{e^{-4x}}$

$$e^{11-2x}(e^{-4x}) = \frac{3}{\cancel{e^{-4x}}}(\cancel{e^{-4x}})$$
$$e^{11-2x-4x} = 3$$
$$e^{11-6x} = 3$$
$$\longrightarrow \quad 11 - 6x = \log_e(3)$$

$$11 - \log_e(3) = 6x$$
$$x = \frac{11 - \log_e(3)}{6}$$

15. $\frac{11 - \log_e(3)}{6}$

16. (1 point) Solve for x if $|7x - 5| < 4$

$$-4 < 7x - 5 < 4$$

$$\longrightarrow \quad -4 < 7x - 5$$
$$-4 + 5 < 7x$$
$$1 < 7x$$
$$\frac{1}{7} < x$$

$$\& \quad 7x - 5 < 4$$
$$7x < 4 + 5$$
$$7x < 9$$
$$x < 9/7$$

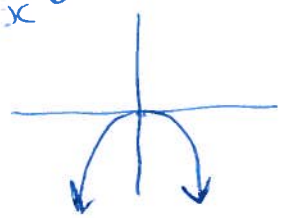
16. $\frac{1}{7} < x < \frac{9}{7}$

Points earned: _____
out of a possible 3 points

17. (5 points) Graph $p(x) = \frac{-3}{5}(x-1)(x-1)(x+2)(x+2)(x^2+9)$

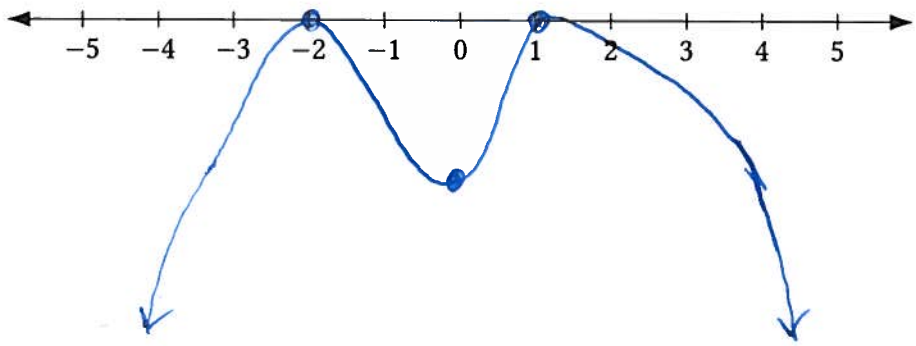
Roots: $1, -2$

Leading term: $-\frac{3}{5}x^6$



$x = 0 \rightarrow p(0) = \frac{-3}{5}(-1)(-1)(2)(2)(9)$

$\rightarrow p(0) < 0$



18. (5 points) Graph $r(x) = \frac{-8(x-1)(x^2+7)}{5(x-3)(x+1)(x+1)}$

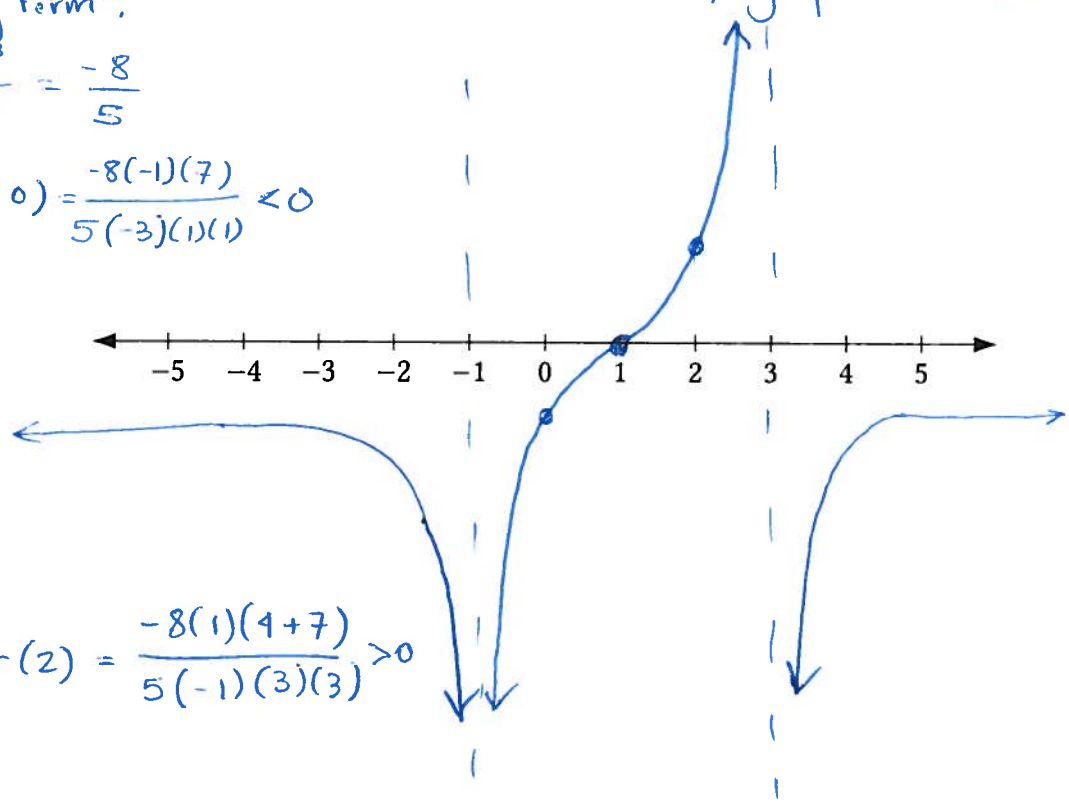
Roots: 1

Asymptotes: 3 & -1

Leading term:

$\frac{-8x^3}{5x^3} = \frac{-8}{5}$

$x = 0 \rightarrow r(0) = \frac{-8(-1)(7)}{5(-3)(1)(1)} < 0$

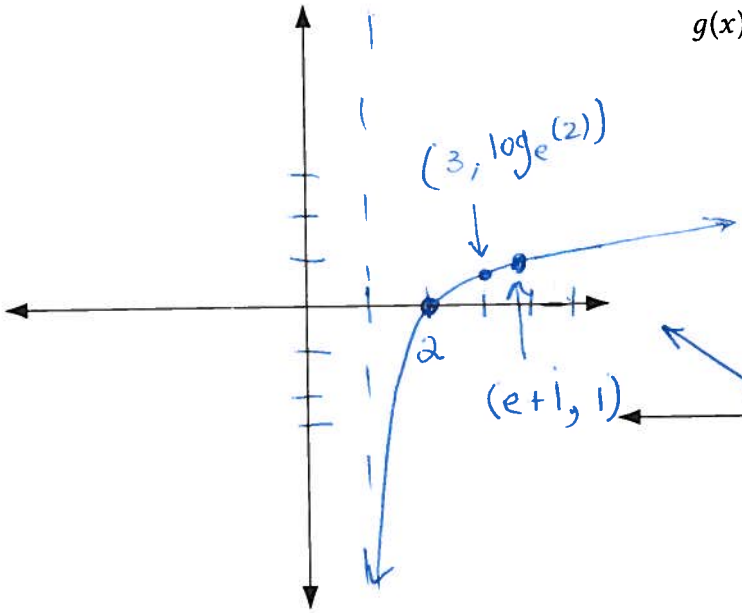


$x = 2 \rightarrow r(2) = \frac{-8(1)(4+7)}{5(-1)(3)(3)} > 0$

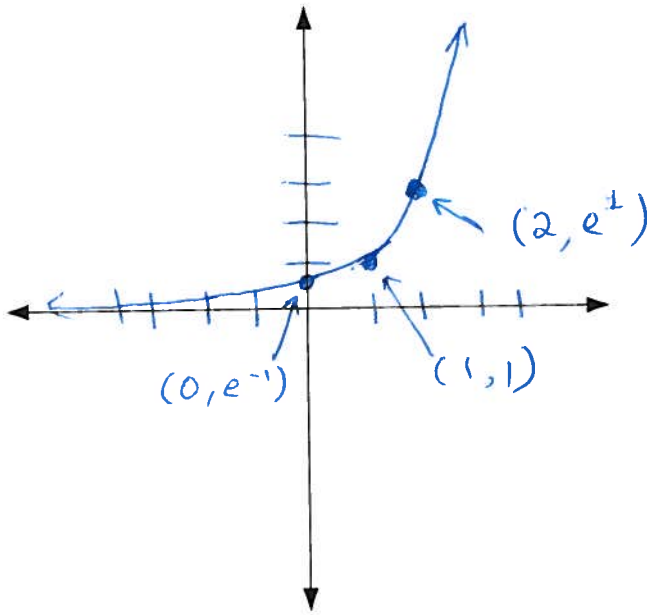
Points earned: _____
out of a possible 10 points

For #22–25, graph the function, draw and label its x- and y-intercepts (if any), its vertical asymptotes (if any), and one additional point.

19. (3 points) $\log_e(x-1)$

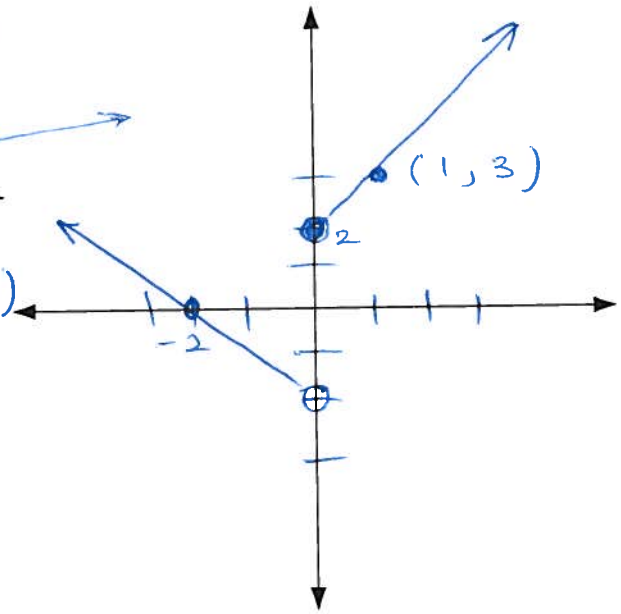


20. (4 points) e^{x-1}



21. (3 points) Graph $g : \mathbb{R} \to \mathbb{R}$ where

$$g(x) = \begin{cases} -x-2 & \text{if } x \in (-\infty, 0) \\ x+2 & \text{if } x \in [0, \infty) \end{cases}$$



Points earned: _____
out of a possible 10 points

22. (a) (1 point) What is the slope of the line $9x - 5y = 1$?

$9x - 1 = 5y$

$\frac{9x-1}{5} = y$

$y = \frac{9x}{5} - \frac{1}{5}$

(a) $9/5$

(b) (2 points) Is there a unique solution to the following system?

$$9x - 5y = 1 \longrightarrow \text{Slope} = 9/5$$
$$-6x + 3y = 17 \longrightarrow \text{Slope} = 2$$

$3y = 17 + 6x$

$y = \frac{17 + 6x}{3}$

$y = \frac{17}{3} + \frac{6x}{3}$

$y = \frac{17}{3} + 2x$

(b) Yes, slopes are different.

23. (2 points) The following system of two linear equations has a unique solution. Find the solution. (Your answer should be a rational number in standard form.)

$$2x + y = 9$$
$$-2x + 3y = 11$$

$2x = 9 - y$

$x = \frac{9-y}{2}$

$-\cancel{2}\left(\frac{9-y}{\cancel{2}}\right) + 3y = 11$

$-9 + y + 3y = 11$

$-9 + 4y = 11$

$4y = 11 + 9$

$4y = 20$

$y = 5$

$x = \frac{9-(5)}{2}$

$x = \frac{4}{2}$

$x = 2$

23. $x = 2$ & $y = 5$

Points earned: _____
out of a possible 5 points